## PREDICTION OF THE SCOUR HOLE GEOMETRY AROUND EXPOSED BRIDGE CIRCULAR-PILE FOUNDATION

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## ABSTRACT

Local scour around bridge piers and abutments, induced by hydraulic deficiencies, is the major cause of bridge failure. Most of the available empirical formulae have been developed based on experimental laboratory tests using uniform sand. However, the bed sediment in the field is to some extent graded. To develop a more reliable formula, well graded sand was used in the experimental flume of this study. In the present paper, the local scour depth around exposed single pile founded in sandy soil was studied experimentally in the laboratory to predict its maximum value. New scour depth prediction equations were developed and compared with some of the previous published equations, other researcher's data, and field data and they were found in very good agreement with them. For complete definition of the scour hole geometry, new equations were developed to estimate the scour hole width at different locations around the pile, thus helping in introducing protection measures for the scour hole. The developed equations are applicable for both clear-water and live-bed scour cases.

KEYWORDS: Scour hole geometry, local scour, circular pile, developed equations.

## **1. INTRODUCTION**

The estimation of scour hole depth around piers foundation became a major concern for engineers because under estimation of the depth of scour endangers the bridge safety whether with shallow or deep foundations. In addition, overestimation of scour depth results in uneconomical design. Therefore, the knowledge of the suitable maximum scour depth for design discharge is essential for proper design of the bridge piers and abutments foundations.

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Scour depth around bridge piers foundations depends on many factors such as the properties of the flow conditions, the pier size and shape, the bed material of the stream bed, etc. Therefore, it is not surprise that the various scour depth prediction equations produce different values.

The local scour may occur when exposed bridge pile foundations obstruct the flow. The local scour in uniform and non-uniform soil may be classified as live-bed or clear-water scour, depending on the bed material transport mode. For clear-water scour, the equilibrium scour depth is reached when the flow is no longer capable of removing bed sediment from the hole (Kwan and Melville, 1994) [1]. The live-bed scour occurs when the scour hole is continuously fed with sediment by the approaching stream (Dey, 1998) [2]. Live-bed scour (also known as scour with sediment transport) occurs when the bed material upstream of bridge piers, piles, or abutments is in motion (Raudkivi, 1998) [3]. The live-bed scour equilibrium is attained over a period of time when the average amount of sediment transported into the scour hole by the approach flow is equal to the average amount of sediment removed from it due to local scour action (Melville, 1983) [4]. Bridge failure due to scour has been investigated by several researchers such as: Richardson et al. (1993) [5], Melville and Sutherland (1988) [6], and Briaud et al. (1999) [7].

Several researchers studied the local scour around bridge pier foundations such as Raudkivi and Ettema (1983) [8], Melville and Sutherland (1988) [6], Johonson (1995) [9], Melville and Raudkivi (1994) [10], Melivlle (1997) [11], Dey (1998) [2], Ettema et. al. (1998) [12], Johonson and Dock (1998) [13], Kandasamy and Melivlle (1998) [14], Melivlle and Chiew (1999) [15], Oliveto (2005) [16], etc. Most of these researchers developed their equations for uniform sand bed mterial. In the present study, a well graded sandy soil was used, thus, developing more reliable equations. Remedial mesures for the scour hole arround bridge foundation have been investigted by some researchers such as Zarrati (2006) [17], Dey (2006) [18], Melville (2006) [19], Unger and Hager (2006)[20], etc.

To introduce any protective measures for the scour hole around bridge piles, the scour hole geometry has to be defined. Therefore, the main objectives of the present research are to develop: i) new empirical formulae that practically simulate the scour depth around circular pile foundation, ii) empirical formulae to predict scour width at different locations around the pile foundation, i.e. to give a complete identification of the scour hole geometry. To achieve these objectives an extensive experimental program was conducted in a large scale re-circulating flume using well graded sandy soil bed, at the Hydraulics and Irrigation Laboratory, Faculty of Engineering, Cairo University. The results of this experimental program are used to extrapolate, the local scour hole depth prediction equations and the scour hole width prediction equations. The proposed equations were verified using the available data from other researchers. Then, the proposed equations were applied on a field problem.

## 2. EXPERIMENTAL SETUP

All the experiments were carried out in a large scale recirculating flume 21.0m long, 2.0m wide, and 0.9m deep as shown in Fig.1, at the Hydraulics and Irrigation Department Laboratory, Faculty of Engineering, Cairo University, Giza, Egypt.

The conducted experiments were at 20cm flow depth for circular pile with three different sizes (5, 10, and 15cm pile diameter). Four different flow conditions ( $F_r$ =0.21, 0.28, 0.32, and 0.36) were applied to each pile size. The maximum pile diameter was 15cm which give a ratio of flume width to pile diameter approximately equal to 13, which means all the pile sizes mentioned above could be used without any sidewall effect on the local scour around piles. The bed material used for these experiments was well-graded sand with a specific gravity  $G_s$ =2.65, median size  $d_{50}$ =0.52mm, geometric standard deviation of particle size  $\sigma_g$ =2.05, and uniformity coefficient U=2.64, Saleh (2003) [21].

## 3. DEVELOPED EQUATIONS FOR SCOUR HOLE GEOMETRY

The scour hole geometry in the present study will be considered as scour hole depth and scour hole width at different locations around the pile. The following paragraphs will present the developed equations for the scour hole depth and width around circular piles.



Fig. 1. Experimental setup plan view (Dimensions distorted)

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## **3.1 Equations for the Scour Hole Depth Prediction**

From the collected data and analysis of the results of this experimental program, two equations were developed to predict the scour hole depth around circular pile. These equations resulted from the regression analysis, using the Sigma Plots Program Package. The two equations could be used for prediction of the maximum local scour hole depth. The first equation was developed by linear regression and the other by power regression of normalized scour depth ( $d_s/b$ ) observed from the experiments. These equations are (the symbol notations are at section 9 at the end of the paper):

i. Linear regression equation with correlation coefficient R = 0.94.

$$\frac{d_s}{b} = K_s K_{\theta} \left( 2.2F_r + 0.12(\frac{y}{b}) + 0.55 \right)$$
(1)

ii. Power regression equation with correlation coefficient R = 0.96.

$$\frac{d_s}{b} = 2.1K_s K_\theta F_r^{0.41} \left(\frac{y}{b}\right)^{0.22} \tag{2}$$

#### **3.2 Equations for the Scour Hole Width Prediction**

The developed equations for the scour hole width were defined at the pile upstream, the pile side, and the pile downstream as follows:

## 3. 2.1 Proposed equation for scour hole width at the pile upstream

The top width of the scour hole at the upstream vicinity of the pile ( $W_s$  up) depends upon a number of factors, which are;  $d_s$ =scour depth, y=approach flow depth, B=the bottom width of the scour hole at upstream,  $\phi$ =Wet angle of repose of the bed material which equals the upstream side slope angle of the scour hole. The regression gives the following equation with a correlation coefficient, R=0.985.

$$W_s up = d_s \cot\phi + B + 0.05 \tag{3}$$

## 3.2.2 Proposed equation for scour hole width at the pile side

The width of the scour hole at pile side ( $W_s$  side) depends upon a number of factors which are;  $d_s$ =scour depth, y=approach flow depth, B=the bottom width of the scour hole at the pile side and  $\phi_{85}$ =85% of wet angle of repose of the bed material (the side slope angle of scour hole at pile side). The regression gives the following equation for the scour hole width with a correlation coefficient, R=0.99.

$$W_s \, side = d_s \, cot \phi_{85} + B + 0.02$$
 (4)

## 3.2.3 Proposed equation for scour hole width at the pile downstream

The scour hole width at pile downstream reach is wider than that at the upstream and sides reaches. Accordingly, the scour hole width at the downstream reach may control the protection of the scour hole area. Therefore, the scour hole width at the pile downstream reach should be estimated. In the present study, the scour hole width at the pile downstream reach was estimated, as a function of Froude number ( $F_r$ ) and approach flow depth pile diameter (y/b). The scour hole width at the pile downstream was obtained by using power regression of normalized scour width ( $W_s$  down/b) observed from the experimental runs. The regression gives the following equation (with correlation coefficient R=0.94):

$$\frac{W_s \, down}{b} = 27.6 \, F_r^{1.51} \left(\frac{y}{b}\right)^{0.26} \tag{5}$$

## 4. VERIFICATION OF THE DEVELOPED EQUATIONS

The developed equations for the scour hole geometry, i.e. depth and width are verified using the previous equations and experimental results of some authors as follows:

## 4.1 Verification of the Scour Hole Depth Equations

The proposed equations for scour depth around single pile are compared with predicted scour depth from some used equations such as: Jain and Fisher (1979),

Froehlich (1987), Melville and Suthreland (1988), Johnson (1992), Richardson (1993), HEC-18 (1995) and Melville (1997) see Saleh (2003) [21]. This comparison was made by two different methods, first by the comparison of the calculated scour depth ratio (ds/b) from different proposed and comparative equations for the same Froude number. Second by comparison of the calculated scour depth ratio (ds/b) from various equations by applying the same conditions used for the measured experimental data, with the measured scour depth ratio.

## 4.1.1 The first method

Figures 2.a - 2.c show the relationship between ds/b and  $F_r$  for pile sizes of 5cm, 10cm and 15cm respectively, and the following observations may be noticed:

- The scour depth predicted using Melville (1997) and Melville & Sutherland (1988) equations is higher than that predicted using the developed Eqs. (1 and 2).
- HEC-18 equations (1995), give over predicted values about 13% difference for 5cm diameter pile, over predicted with small difference of about 4% for 10cm diameter pile, and under predicted very small difference of about 3% for 15cm diameter pile with Eqs. (1 and 2), respectively.
- The scour depth predicted using Richardson's equation (1993) is over estimated with 18% difference for 5cm diameter piles, 10% for 10cm diameter piles, and 4% difference for 15cm diameter piles with Eq. (1) and Eq. (2), respectively.
- Johnson's equation (1992), produced under predicted values about 16% difference for 5cm diameter pile, under predicted value 4% difference for 10cm diameter pile, and over predicted values with 2% and 4% difference for 15cm diameter piles with Eq. (1) and Eq. (2), respectively.
- Froehlech's equation (1987), gives over predicted about 3% difference for 5cm diameter pile, over predicted values with about 10% difference for 10cm diameter pile, and over predicted about 7% difference for 15cm diameter piles with Eq. (1) and Eq. (2), respectively.
- Jain & Fischer's equation (1979), gives high over predicted values with 28% and

27% difference for 5cm diameter piles, over predicted values with 16% and 18% difference for 10cm diameter piles, and over predicted values with 9% and 3% difference for 15cm diameter piles with Eq. (1) and Eq. (2), respectively.

• Coloman (1971) equation gives high under predicted values with 25% and 26% differences for 5 cm diameter piles, high under predicted values with 18% and 19% differences for 10 cm diameter piles, and high under predicted values with 17% and 15% differences for 15cm diameter piles with Eq. (1) and Eq. (2), respectively.



Fig. 2-a. Comparison between calculated normalized scour depth with Froude Number for 5cm pile diameter.



Fig. 2-b. Comparison between calculated normalized scour depth with Froude Number for 10cm pile diameter.



Fig. 2-c. Comparison between calculated normalized scour depth with froude number for 15cm pile diameter.

## 4.1.2 The second method

Figure 3 indicates that the major part of the data falls within the range  $\pm 15\%$  and the following comments are drawn:

- Melville & Sutherland (1988), and Melville (1977), are highly over predicted values out of the selected range  $\pm 15\%$ .
- HEC-18 (1995) equation is in line with the Eqs. (1 and 2) by difference less than 5%.
- Richardson's equation (1993), gives over predicted values with less than 10% difference, for normalized scour depth (ds/b measured)>1.5, and in line with calculated values from Eqs. (1 and 2) with less than 5% difference, for normalized scour depth (ds/b measured )< 1.5.
- Froehlech (1987), and Johnson (1992) equations, give 6% over predicted values with the values calculated from developed Eqs. (1 and 2) for normalized scour depth (ds/b measured ) < 1.75, and 5% under predicted values for normalized scour depth (ds/b measured ) > 1.75.
- Jian & Fischer's equation (1979), gives over predicted values with 10% difference compared to the calculated values from the developed equations.
- Coloman's equation (1971), gives under predicted values, but for normalized scour depth (ds/b measured)>1.5, the calculated results are out of the selected range.



Fig. 3. Comparison between measured and calculated normalized scour hole depth.

## 4.2 Verification of the Developed Equations for the Scour Hole Width

The proposed equations for the scour hole width at the upstream, the sides and downstream of the pile are verified as indicated in the following paragraphs.

## 4.2.1 The scour hole width at the upstream reach

The developed Eq. (3) for scour hole width at the upstream reach of the circular pile best fits the measured data ( $(d_s/b)_{measured}$ ) with less than 5% difference as indicated in Fig. 4. The developed equation was compared to HEC-18 (1995) equation, which is the only one developed for scour hole width around bridge piers.

The proposed equation for scour hole width gives over predicted values with HEC-18 equation. This means that the proposed equation provides more safety for estimating the scour width at the upstream side.

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Fig. 4. Comparison between measured and calculated scour hole width upstream of circular pile.

## 4.2.2 The scour hole width at pile side

Equation (4) is developed from the experimental data of scour hole side widths for circular piles. The proposed equation best fits the measured data with less than 3% over predict as shown in Fig. 5.



Fig. 5. Comparison between measured and calculated scour hole width at pile's side for circular pile.

## 4.2.3 The scour hole width at the downstream

The downstream scour hole width is the largest and it may control the scour hole protection area. Therefore, Eq. (5) is developed to predict the scour hole width at the downstream for circular piles. The proposed equation best fits the measured data with less than  $\pm$  5%, as shown in Fig. 6.



Fig. 6. Comparison between measured and calculated scour hole width downstream of circular pile.

## 5. APPLICATION OF THE DEVLOPED EQUATIONS

The developed Eqs. (1 and 2) are applied to other observed experimental data, field data, and extension data. This application is achieved to check the adequacy of the developed equations to predict the scour depth around bridge pile foundations in the field.

## 5.1 Developed Equations Application to other Experimental Data

The developed Eqs. (1 and 2) for circular piles are applied to two sets of experimental data observed by other researchers. These data are reported for different pile sizes, flow conditions, flow depths and bed materials.

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## 5.1.1 Data reported by Melville and Chiew (1999)

Melville and Chiew (1999) reported a large data observed from different researches. These data were observed from Nan Yang Technological University. These data were approximately good fit the developed Eqs. (1 and 2) with  $\pm$  15% difference as shown in Fig. 7.



Fig. 7. Application of the developed equations to the data of Melville and Chiew (1999).

## 5.1.2 Data observed by Dey (1995)

Dey (1995) reported a data for different circular pile diameters, with different flow conditions, flow depths and bed materials sizes. The developed Eqs. (1 and 2) were applied to this data and gave a good agreement with less than 10% over predicted difference as indicated in Fig. 8.

## 5.2 Developed Equations Application to Field Data

Samuel and Aziz (2001) reported a field data of the scour depth around Aswan bridge piers foundations (from Aswan bridge site in Aswan city, Egypt) see Saleh (2003) [21]. The reported data were average flow depth and discharge at bridge site (y=8.6 and Q=270mm<sup>3</sup>/day), average approach velocity (V=0.66m/s), and piers

diameter (b=7.5 and 2.4m) for the middle and side piers, respectively. The developed Eqs. (1 and 2) are applied to the above mentioned field data and the results are reported in the Table 1. For rounded nose pier, Eq. (1) gives under predicted values with 5% difference for 7.5m width pier, and over predicted values with 18% difference for 2.4m width pier. While Eq. (2) gives under prediction 20% and 16% difference for the 7.5m and 2.4m pier width, respectively.



Fig. 8. Application of the developed equations to the data collected by dey (1995).

b <sub>pier</sub>	d <sub>s field</sub>	d <sub>scalculated</sub>	difference	d <sub>scalculated</sub>	difference	dscalculated	difference
(m)	(m)	$CSU^{*}(m)$	(%)	Eq. (1)	(%)	Eq. (2)	(%)
7.5	8	12.57	+57	7.58	-5	6.32	-21
2.4	3.1	5.75	+85	3.65	+18	2.60	-16

Table 1. Field data application and comparison with Eq. (1 and Eq. 2).

\* Colorado State University equation.

## 6. EXTENSION OF THE DEVELOPED EQUATIONS

The developed Eqs. (1 and 2) are applied to extension data range for  $F_r$  (0.35 to 1.5) and y/b (0.5 to 4). From Fig. 9, the developed equations are in the same trend with the previous comparative equations such as HEC-18 (1995), Richardson (1993), and Jain & Fischer (1979) equations with high correlation. The good agreement with the above mentioned authors is clear for subcritical flow with a difference of 17% and this

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difference increases to about 28.5% in the supercritical flow zone. On the other hand, data of Johnson (1992) doesn't match and behave in a different way especially for  $F_r < 1$ .



Fig. 9. Application of the developed equations to extension data range.

## 7. CONCLUSIONS

For the previous study the following conclusions could be drawn:

- 1. Two Eqs. 1 and 2 for the scour depth around circular pile in sandy soil are developed. These equations are verified with previous reported data, field data, and extension range data, and they give good results with high correlation, and in the same trend of previous equations for scour around piers for different bed material.
- The developed Eqs. (1 and 2) give a good agreement with other observed data by Melville & Chiew (1999) and Dey (1995), with a good correlation R = 0.82 and 0.86 for Melville data and R=0.74 and 0.85 for Dey data respectively for Eqs. (1 and 2).
- 3. The developed Eqs. (1 and 2) for circular pile give very good agreement with less than 5% difference with comparative equations (HEC-18 (1995), Richardson

(1993) and Froehelish (1987)), and predicted with high correlation about R=0.96 ( $R^2 = 0.922$ ).

- 4. The developed equations give good agreement with experimental data from published researches, field, and extension range data. Therefore these equations are recommended to be used for the prediction of scour hole depth around the exposed pile foundations or bridge piers for both clear-water and live-bed scour.
- For the extended data range, the developed equations give a perfect agreement with HEC-18 (1995) and Richardson (1993), while it gives a very good agreement with Johnson (1992) and Jain & Fishers (1979).
- 6. Three Eqs. (3–5) are developed to predict the scour hole width at the upstream, sides and down stream of the pile. For upstream and sides, the equations are in terms of scour depth and soil wet angle. Downstream equations are in terms of Froude number and water depth pile width ratio (y/b).
- 7. The Eqs. (3-5) for prediction of the scour hole width at the pile upstream, sides and downstream reaches for circular pile are validated. The performance of the developed equations is very well, hence the error is less than 5%.

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## SYMBOLS

- d<sub>s</sub> scour hole depth
- b pile diameter.
- $K_s$  pile shape factor (for circular pile,  $K_s = 1$ )
- $K_{\theta}$  pile alignment factor (for single pile,  $K_{\theta} = 1$ )
- K<sub>d</sub> sediment size factor from charts or formulae in previous studies
- K<sub>b</sub> sediment bed factors given in tables
- $F_r$  Froude number = V/(gy)<sup>0.5</sup>
- y the approach flow depth at the upstream
- V water velocity
- g gravity acceleration
- G<sub>s</sub> specific gravity
- d<sub>50</sub> median size

 $\begin{array}{ll} \sigma_g & \text{geometric standard deviation of particle size} \\ U & \text{uniformity coefficient} \\ \text{dsNm} & \text{normalized measured scour depth} \\ \end{array}$ 

dsNc normalized calculated scour depth

# التنبؤ بالأبعاد الهندسية للبيارات الناتجة من النحر الموضعى حول الخوازيق ذات القطاع الدائري لأساسات الكباري

تم فى البحث إستخدام التربة الرملية جيدة التدرج في قاع قناة التجربة حيث أنه الأقرب تمثيلاً للظروف الطبيعية بالموقع، وإستنباط معادلات وضعية بناءً على نتائج التجارب المعملية لتحديد الأبعاد الهندسية لتلك البيارت من حيث العمق والعرض وتم تقييم آداء تلك المعادلات بإستخدام البيانات المتاحة من الدراسات الحقلية أو المعملية بالإضافة إلى بعض المعادلات المنشورة بالمراجع. وكان آداء تلك المعادلات جيداً فى العديد من تلك المقارنات.